UGEB2530 Game and strategic thinking
Solution to Quiz 1

1. (4 marks) Circle all pure Nash equilibria of the games.

   (a) $\begin{pmatrix} (3,-1) & (1,0) \\ (5,2) & (0,-1) \end{pmatrix}$

   (b) $\begin{pmatrix} (1,0) & (6,4) & (3,5) \\ (4,2) & (-1,0) & (2,-1) \end{pmatrix}$

   Solution:

   (a) By the definition of Nash equilibrium, $(1,0)$ and $(5,2)$ are the pure Nash equilibria.

   (b) $(3,5)$ and $(4,2)$ are the pure Nash equilibria.

2. (4 marks) In a game there is a bag which contains 3 red balls and 2 blue balls. Two balls are chosen randomly from the bag without replacement. 3 marks will be given for each red ball and 1 mark will be given for each blue ball.

   (a) Find the probability that the total marks is 4.

   (b) Find the expected total marks of the game.

   Solution:

   (a) The probability is calculated as: $\frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{3}{4} = \frac{3}{5}$.

   (b) The expected total marks is calculated as: $2 \times \frac{2}{5} \times \frac{1}{4} + 4 \times \frac{3}{5} \times \frac{1}{2} + 4 \times \frac{2}{5} \times \frac{3}{4} + 6 \times \frac{3}{5} \times \frac{1}{2} = \frac{22}{5}$.

3. (4 marks) Circle all saddle points of the following game matrices.

   (a) $\begin{pmatrix} 3 & 0 & -2 & 2 \\ 4 & 2 & 1 & 3 \\ 1 & -1 & 0 & 2 \end{pmatrix}$

   (b) $\begin{pmatrix} -4 & 0 & -3 & -3 \\ 0 & -1 & -2 & 1 \\ 3 & -2 & -3 & -5 \\ 2 & 1 & -4 & 2 \end{pmatrix}$

   Solution:

   (a)

					Min
	3	0	-2	2	-2
	4	2	1	3	1
	1	-1	0	2	-1
Max	4	2	1	3	

Thus the saddle point is (R_2, C_3).

(b)

					Min
	-4	0	-3	-3	-4
	0	1	-2	1	-2
	3	-2	-3	-5	-5
	2	1	-4	2	-4
Max	3	1	-2	2	

Thus the saddle point is (R_2, C_3).

4. (8 marks) John calls out a number '1' or '2' and Peter calls out a number '3' or '4' simultaneously. If the sum of the two numbers is even, Peter pays John the sum. If the sum of the two numbers is odd, John pays Peter the sum.

   (a) Write down the payoff matrix for John.

   (b) Suppose John chooses '1' with a probability of 0.4 and Peter chooses '3' with a probability of 0.2. Find the expected payoff of John.

   (c) What is the best strategy of Peter if John chooses '1' with a probability of 0.4?

   (d) Find the strategy of John such that his payoff is fixed no matter how Peter plays.

   (e) Find the value of the game.

Solution:

(a)

	'3'	'4'
'1'	4	-5
'2'	-5	6

(b) The expected payoff is calculated as:

$$\begin{bmatrix} 0.4 & 0.6 \end{bmatrix} \begin{bmatrix} 4 & -5 \\ -5 & 6 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} = 1.$$

(c)

$$\begin{bmatrix} 0.4 & 0.6 \end{bmatrix} \begin{bmatrix} 4 & -5 \\ -5 & 6 \end{bmatrix} = \begin{bmatrix} -1.4 & 1.6 \end{bmatrix}$$

Thus the best strategy for Peter is $(1, 0)$.

(d) The maximin strategy for John is:
$(\frac{11}{20}, \frac{9}{20})$.

(e) The value of the game is:
$\frac{4 \times 6 - (-5) \times (-5)}{4 - (-5) + 6 - (-5)} = -\frac{1}{20}$.

5. (4 marks) Solve the 2×2 game with game matrix

$$\begin{pmatrix} -3 & 4 \\ 3 & 0 \end{pmatrix}$$

that is, find a maximin strategy, a minimax strategies, and the value of the game.

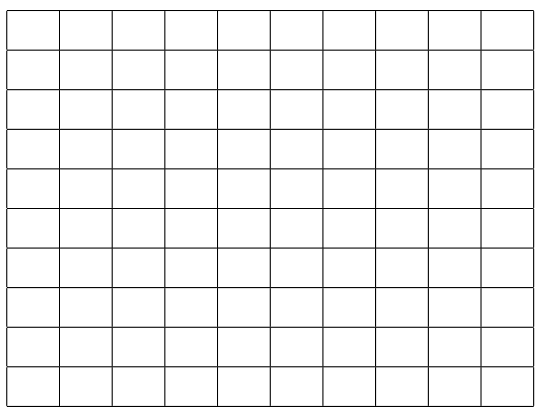
Solution:
The maximin strategy is: $(0.3, 0.7)$.
The minimax strategy is: $(0.4, 0.6)$.
The value of the game is calculated as:

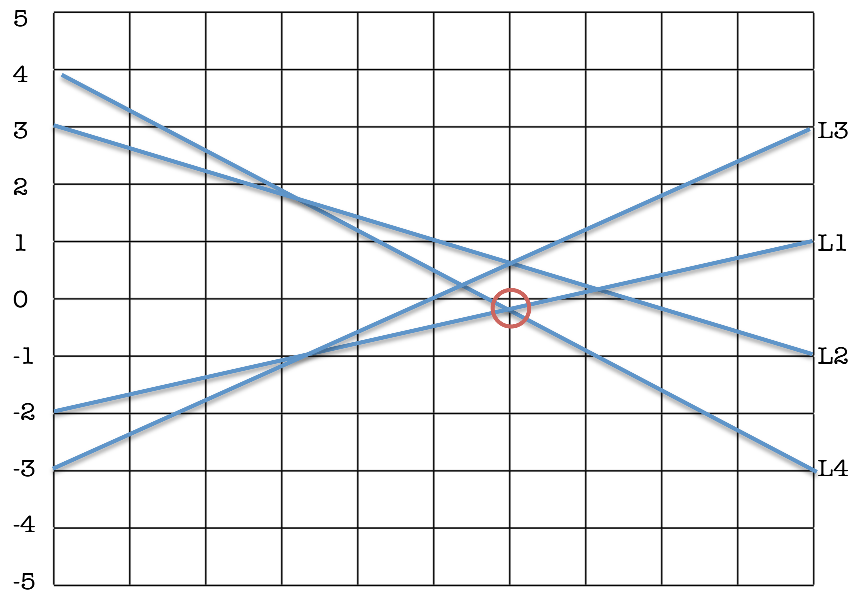
$$\begin{bmatrix} 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} -3 & 4 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = 1.2.$$

6. (6 marks) Solve the zero sum game with the game matrix

$$\begin{pmatrix} 1 & -1 & 3 & -3 \\ -2 & 3 & -3 & 4 \end{pmatrix}$$

solution:





Solution:

Form the picture, the game is reduced to

$$A = \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix}$$

So the the maximin strategy is: $(0.6, 0.4)$.
The minimax strategy is: $(0.7, 0, 0, 0.3)$.
The value of the game is calculated as:

$$\begin{bmatrix} 0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} = -0.2$$

7. (10 marks) Consider the 3×3 game matrix

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

where a is a real number.

(a) Find the value of A if $a \leq 0$.

(b) Suppose $a = 1$.

(i) What is the best strategy of the column player if the row player uses mixed strategy $(0.2, 0.3, 0.5)$?

(ii) Find the maximin strategy for the row player.

(iii) Find the value of A.

Solution:

(a)

				Min
	a	0	0	a
	0	2	0	0
	0	0	3	0
Max	0	2	3	

Thus the saddle point exist whatever $a \leq 0$, and is 0. So the value of the game is 0.

(b) Suppose $a = 1$.

(i)

$$\begin{bmatrix} 0.2 & 0.3 & 0.5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.6 & 1.5 \end{bmatrix}$$

Thus the best strategy is $(1, 0, 0)$..

(ii) Suppose the maximin strategy is $(p, q, 1 - p - q)$ for $0 \leq p, q \leq 1$. The payoff of the row player is then given by:

$$\begin{bmatrix} p & q & 1-p-q \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} p & 2q & 3-3p-3q \end{bmatrix}$$

If this is the maximin strategy we will have $p = 2q$ and $p = 3 - 3p - 3q$ which will imply that:
$p = \dfrac{6}{11}$ and $q = \dfrac{3}{11}$.
Thus the maximin strategy is $(\frac{6}{11}, \frac{3}{11}, \frac{2}{11})$.

(iii) The value of the game is $\dfrac{6}{11}$.